Symbolic calculation of multiparticle Feynman amplitudes

P. Cherzor
Skobeltsyn Institute of Nuclear Physics, Moscow State University
Moscow 119899, Russia

Abstract

Using the newly modified method developed for symbolic evaluation of Feynman amplitudes we examine two processes $2 \to 2$ (including a case of Majorana fermions) at a tree level. Constructing special polarization basis for spinor particles, we obtain compact expressions for helicity amplitudes. We present the regular way for simplification of evaluated symbolic expressions.

Quantum amplitudes corresponded to Feynman diagrams depend on momenta of particles and their polarizations. One can describe polarized Dirac spinors of fermion particles in terms of helicity spinors as far as for massless fermions helicity and chirality are connected [1]. It was developed in [2], [3] the method to calculate polarized matrix elements (polarized squared amplitudes) through helicity amplitudes. A detailed description of spin formalism and helicity spinors use one can find e.g. in [4]. The amplitude is expressed as a product of Dirac γ-matrices ended by two spinors — *fermion string*. The contribution of a fermion string to the Feynman amplitude can be rewritten through the trace:

$$\bar{w}(p_2, \lambda_2)\Gamma\, w'(p_1, \lambda_1) = tr(\Gamma\, w \otimes \bar{w}'), \tag{1}$$

where Γ is some product of γ-matrices.

In [5] it was constructed the special spinor basis, which allows to represent tensor products $w \otimes \bar{w}'$:

$$\begin{aligned}
u(p,\lambda) \otimes \bar{u}(p',\lambda') &= N(m+\hat{p}) \begin{pmatrix} 1-\gamma_5 & (1-\gamma_5)\hat{\eta}^* \\ -(1+\gamma_5)\hat{\eta} & 1+\gamma_5 \end{pmatrix} \hat{k}(m'+\hat{p}')\,; && (2)\\
u(p,\lambda) \otimes \bar{v}(p',\lambda') &= N(m+\hat{p}) \begin{pmatrix} (1-\gamma_5)\hat{\eta}^* & 1-\gamma_5 \\ 1+\gamma_5 & -(1+\gamma_5)\hat{\eta} \end{pmatrix} \hat{k}(m'-\hat{p}')\,; && (3)\\
v(p,\lambda) \otimes \bar{u}(p',\lambda') &= N(m-\hat{p}) \begin{pmatrix} -(1+\gamma_5)\hat{\eta} & 1+\gamma_5 \\ 1-\gamma_5 & (1-\gamma_5)\hat{\eta}^* \end{pmatrix} \hat{k}(m'+\hat{p}')\,; && (4)\\
v(p,\lambda) \otimes \bar{v}(p',\lambda') &= N(m-\hat{p}) \begin{pmatrix} 1+\gamma_5 & -(1+\gamma_5)\hat{\eta} \\ (1-\gamma_5)\hat{\eta}^* & 1-\gamma_5 \end{pmatrix} \hat{k}(m'-\hat{p}')\,. && (5)
\end{aligned}$$

Here $N = 1/[4\sqrt{(pk)(p'k)}]$, $m = \sqrt{p^2}$, $m' = \sqrt{p'^2}$, and polarizations' dependence follows the rule: $\begin{pmatrix} ++ & +- \\ -+ & -- \end{pmatrix}$. The products $w \otimes \bar{w}'$ with C-conjugation can be derived by means of relations:

$$u(p,\pm1) = v(p,\pm1)^c,\ \ v(p,\pm1) = u(p,\pm1)^c, \tag{6}$$

$$\bar{u}(p,\pm 1) = \overline{v(p,\pm 1)^c},\ \ \bar{v}(p,\pm 1) = \overline{u(p,\pm 1)^c}. \tag{7}$$

In the proposed method there were introduced three 4-vectors in Minkowsky space: $(k,\ \eta,\ \eta^*)$ the same set of auxiliary vectors was introduced for arbitrary number of fermion strings in the amplitudes. They satisfy the relations: $(\eta,\eta^*) = -1/2$, $\eta^2 = 0$, $(\eta^*)^2 = 0$, $(\eta,k) = 0$, $k^2 = 0$.

The main features of the method we had used:

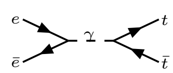
1. it gives formulae uniformly applicable in massive and massless cases, for Dirac and Majorana fermions;

2. it can be applied to interactions with the fermion number violation;

3. it yields more compact symbolic information on helicity amplitudes, especially as far as the variety of polarization vectors is reduced to three: η, η^*, k.

Examples of symbolic evaluation of helicity amplitudes

Let us consider the processes $2 \to 2$ at the tree level as an example of evaluation helicity amplitudes. Consider γ^*-exchange diagram for electron-positron scattering to the quark and anti-quark $e\overline{e} \to t\ \overline{t}$.



diagr.1

In the lowest PT order tree level corresponding quantum amplitude contains the fermion string:

$$o(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = \overline{v}_{\overline{e}}(p_1,\lambda_1)\ \gamma_\mu\ u_e(p_2,\lambda_2)\ \overline{u}_t(q_1,\sigma_1)\ \gamma^\mu\ v_{\overline{t}}(q_2,\sigma_2)\ 1/s,$$

where p_i, q_j are particles momenta, λ_i, σ_j are polarizations. $1/s$ is the propagator, $s = (p_1+p_2)^2$. One can consider electron and positron as massless, m is the mass of $t, \overline{t}$.

One can connect fermion legs in a three topologically different ways, each case gives the definite trace structure in the fermion string correspondingly:

$$o_1(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = Tr\ \{\gamma_\mu\ u_e(p_2,\lambda_2)\ \ \overline{v}_{\overline{e}}(p_1,\lambda_1)\}\ Tr\ \{\gamma^\mu\ v_{\overline{t}}(q_2,\sigma_2)\ \overline{u}_t(q_1,\sigma_1)\}\ 1/s,$$

$$o_2(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = Tr\ \{\gamma_\mu\ u_e(p_2,\lambda_2)\ \overline{u}_t(q_1,\sigma_1)\ \gamma^\mu\ v_{\overline{t}}(q_2,\sigma_2)\ \overline{v}_{\overline{e}}(p_1,\lambda_1)\}\ 1/s,$$

$$o_3(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = -Tr\ \{(-\gamma_\mu)\ \ v^c_{\overline{e}}(p_1,\lambda_1)\ \overline{u}_t(q_1,\sigma_1)\ \gamma^\mu\ u^c_{\overline{t}}(q_2,\sigma_2)\ \overline{u}^c_e(p_2,\lambda_2)\}\ 1/s.$$

Third expression is the case of reversed spinors' order in a trace and of presence of spinor products of C-conjugated spinors. The details of helicity amplitudes processing algorithm, which includes a possibility to reverse the spinors' order and the appearance of C-conjugated spinors in a trace, will be described in [6]. One must replace the vertices according to [7]. It was described in [5] how to construct the spinor products. For C-conjugated spinors one must use the substitutions (6), (7) in (2)-(5). We give an example of the result of the traces evaluation for fermion string for the set of helicities (1,-1,-1,1), here and bellow in this section we define the common normalization factor $N = 1/16\sqrt{(p_1,k)(p_2,k)(q_1,k)(q_2,k)}$, $N_1 = N/s$:

$$o_1(1,-1,-1,1) = 8\,N_1\,[-(k,p_1)\,(k,p_2)\,(q_1,q_2)\ -m^2\,(k,p_1)\,(k,p_2)$$

$$+(k,p_1)\,(k,q_2)\,(q_1,p_2)\,-i\,(k,p_1)\,e_{\mu\nu\sigma\rho}k^\mu {p_2}^\nu {q_1}^\sigma {q_2}^\rho$$
$$+(k,p_2)\,(k,q_1)\,(p_1,q_2)\,-(k,q_1)\,(k,q_2)\,(p_1,p_2)\,-i\,(k,q_1)\,e_{\mu\nu\sigma\rho}k^\mu {p_1}^\nu {p_2}^\sigma {q_2}^\rho],$$
$$o_2(1,-1,-1,1)=8\,N_1\,[-(k,p_1)\,(k,p_2)\,(q_1,q_2)\,-m^2\,(k,p_1)\,(k,p_2)$$
$$+(k,p_1)\,(k,q_2)\,(q_1,p_2)\,-i\,(k,p_1)\,e_{\mu\nu\sigma\rho}k^\mu {p_2}^\nu {q_1}^\sigma {q_2}^\rho$$
$$+(k,p_2)\,(k,q_1)\,(p_1,q_2)\,-(k,q_1)\,(k,q_2)\,(p_1,p_2)\,-i\,(k,q_1)\,e_{\mu\nu\sigma\rho}k^\mu {p_1}^\nu {p_2}^\sigma {q_2}^\rho].$$

Initial formula before the trace evaluation for $o_3(1,-1,-1,1)$ contains both η and η^* in one term.

$$o_3(1,-1,-1,1)=4\,N_1\,[-(k,p_1)\,(k,p_2)\,(q_1,q_2)\,-2(k,p_1)\,(k,p_2)\,(\eta,q_1)\,(\eta^*,q_2)$$
$$+2(k,p_1)\,(k,p_2)\,(\eta^*,q_1)\,(\eta,q_2)\,-m^2\,(k,p_1)\,(k,p_2)+(k,p_1)\,(k,q_2)\,(q_1,p_2)$$
$$-2(k,p_1)\,(k,q_2)\,(\eta,p_2)\,(\eta^*,q_1)+2(k,p_1)\,(k,q_2)\,(\eta^*,p_2)\,(\eta,q_1)$$
$$-2i\,(k,p_1)\,(\eta^*,q_1)\,e_{\mu\nu\sigma\rho}k^\mu {p_2}^\nu {q_2}^\sigma \eta^\rho+(k,p_2)\,(k,q_1)\,(p_1,q_2)\,+2(k,p_2)\,(k,q_1)\,(\eta,p_1)\,(\eta^*,q_2)$$
$$-2(k,p_2)\,(k,q_1)\,(\eta^*,p_1)\,(\eta,q_2)+2i\,(k,p_2)\,(\eta,q_2)\,e_{\mu\nu\sigma\rho}k^\mu {p_1}^\nu {q_1}^\sigma (\eta^*)^\rho$$
$$+2im^2\,(k,p_2)\,e_{\mu\nu\sigma\rho}k^\mu {p_1}^\nu \eta^\sigma (\eta^*)^\rho-(k,q_1)\,(k,q_2)\,(p_1,p_2)$$
$$-2(k,q_1)\,(k,q_2)\,(\eta,p_1)\,(\eta^*,p_2)\,+2(k,q_1)\,(k,q_2)\,(\eta^*,p_1)\,(\eta,p_2)$$
$$+2i\,(k,q_1)\,(\eta^*,p_1)\,e_{\mu\nu\sigma\rho}k^\mu {p_2}^\nu {q_2}^\sigma \eta^\rho-2i\,(k,q_2)\,(\eta,p_2)\,e_{\mu\nu\sigma\rho}k^\mu {p_1}^\nu {q_1}^\sigma\;(\eta^*)^\rho],$$

For all possible ways of connection of fermion legs one can select definite expression for fermion string for each set of helicities $(\lambda_1,\lambda_2,\sigma_1,\sigma_2)$ so, that expressions containing terms with both η and η^* will be always excluded [6]. Doing *this choice before the trace evaluation allows ones to reduce the time and resources during trace evaluation*, because so we reduce the variety of symbolic structures in the result of trace evaluation. Here $o_1(1,-1,-1,1)$ and $o_2(1,-1,-1,1)$ satisfy the criteria of non-appearing terms with both η and η^*.

There exist a possibility to simplify further *using and processing helicity amplitudes*. One can define, the orientation of the helicity basis auxiliary vectors in the following way: $k^\mu=(k',0,0,k')$, $\eta^\mu=(0,1/2,i/2,0)$, $(\eta^*)^\mu=(0,1/2,-i/2,0)$. The expressions for fermion strings remains invariant to the Lorentz transformations of initial and final particles' momenta. Only k,η,η^* is fixed. One can express all terms containing $e_{\mu\nu\sigma\rho}$ in the results of evaluation of fermion strings as a products of scalar products using a component-wise representation of all vectors

$$e_{\mu\nu\sigma\rho}\,p^\mu q^\nu P^\sigma Q^\rho=\sum_{i,j,l,m=0,\dots 3}^{i\neq j\neq l\neq m}(p,e_i)(q,e_j)(P,e_l)(Q,e_m)\,e^{ijlm},\quad e^{0123}=1, \tag{8}$$
$$(p,q)=(p,e_0)(q,e_0)-(p,e_1)(q,e_1)-(p,e_2)(q,e_2)-(p,e_3)(q,e_3)$$

and recognizing a scalar products in this component-wise representation.

A general form of the result for fermion string is a sum over all possible combinations of any scalar products of two external momenta and any scalar products of one of external momenta and one of basis vectors k,η,η^*. Then one apply the identity (here $\{p,q\}$ are external momenta, $\widetilde{k}$ is temporary additional auxiliary vector, satisfying $(k,\widetilde{k})=1,\ (\eta,\widetilde{k})=0,\ \widetilde{k}^2=0$):

$$(p,q)=-2[(p,\eta)(q,\eta^*)+(p,\eta^*)(q,\eta)]+(p,\widetilde{k})(q,k)+(p,k)(q,\widetilde{k}).$$

After applying this identity $\widetilde{k}$ cancel themselves inside any helicities' fermion string and do not appear in results due to the symbolic structure of the results for helicity amplitudes we regard in this article, because for any different external momenta p, q, P, Q the scalar products of any two external momenta appears only as a part of the following combination:

$$(p,k)(q,k)(Q,P)+(P,k)(Q,k)(p,q)-(p,k)(Q,k)(q,P)-(q,k)(P,k)(p,Q).$$

Together with terms containing scalar products arising from convolutions (8), scalar products of external momenta expressed through scalar products with η and η^* contribute to the combinations of scalar products of one of external momenta and one of basis vectors k,η,η^*, so that symbolic structure of external momenta (p,q) can be completely removed from the results. Non-zero results for all sets of helicities are the following (expressions are the same for all ways of connection of fermion legs):

$$o'(1,-1,-1,-1)=16m(k,p_2)\,[\,(k,p_1)\,(\eta^*,q_1)+(k,p_1)\,(\eta^*,q_2)-(k,q_1)\,(\eta^*,p_1)-(k,q_2)\,(\eta^*,p_1)\,],$$

$$o'(-1,1,-1,-1)=16m(k,p_1)\,[(k,p_2)\,(\eta^*,q_1)+(k,p_2)\,(\eta^*,q_2)-(k,q_1)\,(\eta^*,p_2)-(k,q_2)\,(\eta^*,p_2)\,],$$

$$o'(1,-1,1,-1)=8[4(k,p_1)\,(k,p_2)\,(\eta,q_1)\,(\eta^*,q_2)-m^2\,(k,p_1)\,(k,p_2)-4(k,p_1)\,(k,q_1)\,(\eta,p_2)\,(\eta^*,q_2)$$
$$-4(k,p_2)\,(k,q_2)\,(\eta^*,p_1)\,(\eta,q_1)\,+4(k,q_1)\,(k,q_2)\,(\eta^*,p_1)\,(\eta,p_2)\,],$$

$$o'(-1,1,1,-1)=8[4(k,p_1)\,(k,p_2)\,(\eta,q_1)\,(\eta^*,q_2)\,-m^2\,(k,p_1)\,(k,p_2)$$
$$-4(k,p_1)\,(k,q_2)\,(\eta^*,p_2)\,(\eta,q_1)\,-4(k,p_2)\,(k,q_1)\,(\eta,p_1)\,(\eta^*,q_2)\,+4(k,q_1)\,(k,q_2)\,(\eta,p_1)\,(\eta^*,p_2)\,],$$

$$o'(1,-1,-1,1)=8[4(k,p_1)\,(k,p_2)\,(\eta^*,q_1)\,(\eta,q_2)-m^2\,(k,p_1)\,(k,p_2)-4(k,p_1)\,(k,q_2)\,(\eta,p_2)\,(\eta^*,q_1)$$
$$-4(k,p_2)\,(k,q_1)\,(\eta^*,p_1)\,(\eta,q_2)\,+4(k,q_1)\,(k,q_2)\,(\eta^*,p_1)\,(\eta,p_2)\,],$$

$$o'(-1,1,-1,1)=8[4(k,p_1)\,(k,p_2)\,(\eta^*,q_1)\,(\eta,q_2)\,-m^2\,(k,p_1)\,(k,p_2)$$
$$-4(k,p_1)\,(k,q_1)\,(\eta^*,p_2)\,(\eta,q_2)\,-4(k,p_2)\,(k,q_2)\,(\eta,p_1)\,(\eta^*,q_1)\,+4(k,q_1)\,(k,q_2)\,(\eta,p_1)\,(\eta^*,p_2)\,],$$

$$o'(1,-1,1,1)=16m(k,p_1)\,[-(k,p_2)\,(\eta,q_1)\,-(k,p_2)\,(\eta,q_2)\,+(k,q_1)\,(\eta,p_2)\,+(k,q_2)\,(\eta,p_2)\,],$$

$$o'(-1,1,1,1)=16m(k,p_2)\,[-(k,p_1)\,(\eta,q_1)\,-(k,p_1)\,(\eta,q_2)\,+(k,q_1)\,(\eta,p_1)\,+(k,q_2)\,(\eta,p_1)\,].$$

For simplicity we drop factor N_1: $o'(\lambda_1,\lambda_2,\sigma_1,\sigma_2)=o(\lambda_1,\lambda_2,\sigma_1,\sigma_2)/N_1$ in the above expressions. Here we observe that scalar products of two external momenta do not appear in the result.

One may represent the results in a shorter form. Collect all scalar products to $e_{\mu\nu\sigma\rho}$-structures using the identities:

$$\Delta(p)=i/2\,(p,k)=\,e_{\mu\nu\sigma\rho}k^\mu\,p^\nu\eta^\sigma(\eta^*)^\rho, \tag{9}$$

$$\Lambda(p,q)=i[(\eta,p)\,(q,k)-(\eta,q)\,(p,k)]=\,e_{\mu\nu\sigma\rho}k^\mu\,p^\nu q^\sigma\eta^\rho, \tag{10}$$

$$\Lambda^*(p,q)=i[(p,k)\,(\eta^*,q)-(p,\eta^*)\,(q,k)]=\,e_{\mu\nu\sigma\rho}k^\mu\,p^\nu q^\sigma(\eta^*)^\rho, \tag{11}$$

Non-zero results for helicity amplitudes became the following:

$$o(1,-1,-1,-1)=N_1\,32\,m\,\Delta(p_2)\;\;\Lambda^*((q_1+q_2),p_1)\,,$$

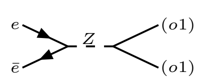
$$o(-1,1,-1,-1)=N_1\,32\,m\,\Delta(p_1)\;\Lambda^*((q_1+q_2),p_2),$$

$$o(1,-1,1,-1)=N_1\,32\,[\,\Lambda^*(p_1,q_2)\;\Lambda(p_2,q_1)+\,m^2\Delta(p_1)\,\Delta(p_2)],$$

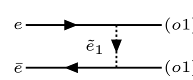
$$o(-1,1,1,-1) = N_1\,32\,[\,\Lambda^*(p_2,q_2)\;\Lambda(p_1,q_1) +\; m^2\Delta(p_1)\,\Delta(p_2)],$$
$$o(1,-1,-1,1) = N_1\,32\,[\Lambda^*(p_1,q_1)\;\Lambda(p_2,q_2) +\; m^2\Delta(p_1)\,\Delta(p_2)],$$
$$o(-1,1,-1,1) = N_1\,32\,[\Lambda^*(p_2,q_1)\;\Lambda(p_1,q_2) +\; m^2\Delta(p_1)\,\Delta(p_2)],$$
$$o(1,-1,1,1) = N_1\,32\,m\,\Delta(p_1)\,\Lambda((q_1+q_2),p_2)\,,$$
$$o(-1,1,1,1) = N_1\,32\,m\,\Delta(p_2)\,\Lambda((q_1+q_2),p_1).$$

Our result for helicity amplitudes for selected diagram do not contradict to the results obtained earlier by other researchers (compare e.g. [8]). Although we did use helicity basis that differs from one of [8], so the straightforward symbolic comparison fails. Indeed the value of the helicity amplitude depends on the method with which it was built. Amplitude in general case could have a complex part. The values those can be compared are polarized matrix elements (squared amplitudes). On the initial stage of our research we did not apply detailed comparison methods to check agreement with other researches done. Instead we concentrated ourself, first, on check the correctness of the results comparing to non-polarized matrix element that evaluates in a well-known way and the correctness under transformations of helicity basis, spinor substitutions due to C-conjugation symmetry, changes trace structure applied. To make a numerical tests we assigned the numerical values (allowed by conservation and Minkowsky space symmetry laws) to particles' momenta and masses.

As an example with Majorana fermions we had examined the process $e\overline{e} \to$ *neutralino pair* in the lowest order tree level with the same procedure of traces optimization, here we also considered electron and positron as massless. This is a process that includes particles neutralino and anti-neutralino, they transfer one into the other by C-conjugation operation.



diagr.1



diagr.2

One can write amplitude for the diagram with Z-boson at s-channel in a following form:

$$f(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = \overline{v}_{\overline{e}}(p_1,\lambda_1)\,\Gamma_\mu\,u_e(p_2,\lambda_2)\,\overline{u}_{\widetilde{\chi}^0_1}(q_1,\sigma_1)\,\Gamma'^\mu\;v_{\widetilde{\chi}^{0\,(c)}_1}(q_2,\sigma_2)\;1/(s-{m_Z}^2),$$

where

$$\Gamma_\mu = \cos(2\theta_w)\,\gamma_\mu\,(1-\gamma^5) - 2(\sin\theta_w)^2\,\gamma_\mu\,(1+\gamma^5),\;\Gamma'_\mu = c_1\,\gamma_\mu\,\gamma^5$$

is the vertices structure. m_Z is the Z-bozon mass. For another diagram with s-electron at t-channel:

$$v(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = \overline{v}_{\overline{e}}(p_1,\lambda_1)\,\Pi\,v_{\widetilde{\chi}^{0\,(c)}_1}(q_2,\sigma_2)\,\overline{u}_{\widetilde{\chi}^0_1}(q_1,\sigma_1)\,\Pi'\,u_e(p_2,\lambda_2)\;1/(t-{m_{\tilde{e}}}^2).$$

$\Pi = c_2\,(1{+}\gamma^5)$, $\Pi' = c_2\,(1{-}\gamma^5)$ are vertices. c_1, c_2 depend on SUSY parameters, t – Mandelstam variable, $m_{\tilde{e}}$ – selectron mass.

In this case one have a possibility to make a fermion lines conjunction of Feynman diagrams in three topologically inequivalent ways for both diagrams. We apply formulae:

$$f_1(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = Tr\;\{\Gamma_\mu\,u_e(p_2,\lambda_2)\,\overline{u}_{\widetilde{\chi}^0_1}(q_1,\sigma_1)\,\Gamma'^{\mu}\;v_{\widetilde{\chi}^{0\,(c)}_1}(q_2,\sigma_2)\,\overline{v}_{\overline{e}}(p_1,\lambda_1)\}\;1/(s-{m_Z}^2),$$

$$f_2(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = Tr\ \{\Gamma_\mu\, u_e(p_2,\lambda_2)\,\overline{v}_{\overline{e}}(p_1,\lambda_1)\}\ Tr\ \{\Gamma'^\mu\, v_{\widetilde{\chi}_1^{0\,(c)}}(q_2,\sigma_2)\,\overline{u}_{\widetilde{\chi}_1^0}(q_1,\sigma_1)\}\ 1/(s-{m_Z}^2),$$

$$f_3(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = -Tr\ \{\widetilde{\Gamma}_\mu\, v^c_{\overline{e}}(p_1,\lambda_1)\,\overline{u}_{\widetilde{\chi}_1^0}(q_1,\sigma_1)\,\Gamma'^\mu\, v_{\widetilde{\chi}_1^{0\,(c)}}(q_2,\sigma_2)\,\overline{u}^c_e(p_2,\lambda_2)\}\ 1/(s-{m_Z}^2),$$

where one obtains $\widetilde{\Gamma}_\mu$ form Γ_μ using rules [7]. $N_2 = N/(s-{m_Z}^2)$.

$$v_1(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = Tr\ \{\Pi\ v_{\widetilde{\chi}_1^{0\,(c)}}(q_2,\sigma_2)\,\overline{u}_{\widetilde{\chi}_1^0}(q_1,\sigma_1)\ \Pi'\, u_e(p_2,\lambda_2)\ \overline{v}_{\overline{e}}(p_1,\lambda_1)\}\ 1/(t-{m_{\tilde{e}}}^2),$$

$$v_2(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = Tr\ \{\Pi\ v_{\widetilde{\chi}_1^{0\,(c)}}(q_2,\sigma_2)\,\overline{v}_{\overline{e}}(p_1,\lambda_1)\}\ Tr\ \{\Pi'\ u_e(p_2,\lambda_2)\,\overline{u}_{\widetilde{\chi}_1^0}(q_1,\sigma_1)\}\ 1/(t-{m_{\tilde{e}}}^2),$$

$$v_3(\lambda_1,\lambda_2,\sigma_1,\sigma_2) = -Tr\ \{\Pi\ v^c_{\overline{e}}(p_1,\lambda_1)\ \overline{u}_{\widetilde{\chi}_1^0}(q_1,\sigma_1)\ \Pi'\ u_e(p_2,\lambda_2)\ \overline{v}^c_{\widetilde{\chi}_1^{0\,(c)}}(q_2,\sigma_2)\}\ 1/(t-{m_{\tilde{e}}}^2).$$

Reversing a direction of fermion line in $v_3(\lambda_1,\lambda_2,\sigma_1,\sigma_2)$ does not change a vertices structure because vertices do not contain γ_μ but γ_5. In cases of C-conjugated spinors we had implied the formulae (2)-(5) with substitutions (6), (7). We had find the same symbolic structures after trace evaluation as in non-Majorana case, and had applied the same procedure of optimization symbolic results. We show here results (non-zero expressions) one obtains after simplification for helicity amplitudes fermion strings for the diagram with Z-boson at s-channel:

$$f(1,-1,-1,-1) = N_2\,64\,c_1\ \cos(2\theta_w)\,m\,\Delta(p_2)\,\Lambda^*(q_2-q_1,p_1),$$

$$f(-1,1,-1,-1) = -N_2\,64\,(2\ \sin^2\theta_w)\,m\,\Delta(p_1)\,\Lambda^*(q_2-q_1,p_2),$$

$$f(1,-1,1,-1) = N_2\,64\,c_1\ \cos(2\theta_w)\,[\Lambda^*(p_1,q_2)\,\Lambda(p_2,q_1) - m^2\,\Delta(p_1)\,\Delta(p_2)],$$

$$f(-1,1,1,-1) = -N_2\,64\,c_1\,(2\ \sin^2\theta_w)\,[\Lambda^*(p_2,q_2)\,\Lambda(p_1,q_1) - m^2\,\Delta(p_1)\,\Delta(p_2)],$$

$$f(1,-1,-1,1) = N_2\,64\,c_1\ \cos(2\theta_w)\,[-\Lambda^*(p_1,q_1)\,\Lambda(p_2,q_2) + m^2\,\Delta(p_1)\,\Delta(p_2)],$$

$$f(-1,1,-1,1) = -N_2\,64\,c_1\,(2\ \sin^2\theta_w)\,[-\Lambda^*(p_2,q_1)\,\Lambda(p_1,q_2) + m^2\,\Delta(p_1)\,\Delta(p_2)],$$

$$f(1,-1,1,1) = N_2\,64\,c_1\ \cos(2\theta_w)\,m\,\Delta(p_1)\,\Lambda(q_2-q_1,p_2),$$

$$f(-1,1,1,1) = -N_2\,64\,c_1\,(2\ \sin^2\theta_w)\,m\,\Delta(p_2)\,\Lambda(q_2-q_1,p_1),$$

and for the diagram with selectron at t-channel:

$$v(1,-1,-1,-1) = N_3\,64\,{c_2}^2\,m\,\Delta(p_2)\,\Lambda^*(q_2,p_1),\quad v(1,-1,1,-1) = N_3\,64\,{c_2}^2\,\Lambda^*(p_1,q_2)\,\Lambda(p_2,q_1),$$

$$v(1,-1,-1,1) = N_3\,64\,{c_2}^2\,m^2\,\Delta(p_1)\,\Delta(p_2),\quad v(1,-1,1,1) = N_3\,64\,{c_2}^2 m\,\Delta(p_1)\,\Lambda(q_1,p_2).$$

Variable m is the neutralinos' mass, $N_3 = N/(t-{m_{\tilde{e}}}^2)$. One can see that chiral symmetry in vertices factors gives the influence on the structure of helicity amplitudes.

Conclusions

We have considered two examples of evaluation helicity amplitudes for the tree level for the processes $2 \rightarrow 2$ (including the case of Majorana external particles).

The key point of our approach is the fixation of three polarization basis vectors — helicity basis — the same for each spinor pair. In case of use massive spinors if to be precise one should keep the orientation of the polarization vectors that is in general not coincide with the directions of ones for massless spinors. In our consideration we left the directions of polarization vectors as for massless case indicating the mass presence outside of polarization vectors part. See (2)-(5). The above approach leads to significant simplifications in case of multi-particle

production. The recipe of traces structure optimization before traces evaluation proposed in [5] was implied successfully (see [6]).

After fixation the helicity basis the further optimization of symbolic structure of the results is achieved for regarded scattering processes. At the first step of optimization the results had been written in a form of scalar products. Symbolic results for pre-optimized helicity amplitudes obtained in this form do not depend on the order of connection of external particles lines. This feature serves for verification the correctness of symbolic expressions for helicity amplitudes.

Scalar products in the considered helicity fermion strings could be completely factorized in the form of special $\Delta, \Lambda, \Lambda^*$ symbolic structures. The last could be also a regular feature, possibly it remains in the scattering $2 \to n$ at least in some cases of the traces' structure. A reason to think so is: at least in the case of real particles one may treat the fermion string for the diagram as a number of traces: $Tr\ \{\Gamma\, w_i(p_1, \lambda_1)\ \overline{w}_j(p_2, \lambda_2)\}$, where Γ is a vertex (or a construction that consists of some vertices and lines either internal (propagators of virtual particles) or external (boson lines e.g.)) and w are the spinors, and each such trace has a form, we can observe in the case $2 \to 2$ in a sense of spinors structure, but all non-trivial symbolic structures (containing spinor particles' polarization vectors) arises exactly from spinor products. The difficulties may arise when we will deal with: virtual particles, exotic expressions for vertices, external boson lines, so that the Γ structure will be complicate.

The procedure implied for symbolic optimization of regarded processes' helicity amplitudes could be extended to the case of more complicated processes. Possibly it may occur rather economical to operate with results for helicity amplitudes in symbolic form, then to store all terms for usual non-polarized cross-sections for the calculations in the case of scattering $2 \to n$, where $n > 4$.

As the next step it could be considered how to apply more advanced tests for our helicity amplitudes' results and comparison methods with the results obtained by the other reseachers in application for calculation polarized matrix elements for the elementary particles' scattering processes.

Acknowledgments

This work is supported by grants: RFFI 01-02-16710, INTAS CERN99-0377.

Sincere thanks to prof. A. Pukhov for fruitful discussions and important support and to prof. V. Ilyin for discussions and valuable support during work on project at Skobeltsyn Institute of Nuclear Physics, Moscow State University.

References

[1] C. Itzykson and J. Zuber, "Quantum Field Theory", McGraw-Hill, N.-Y., 1980.

[2] R. Kleiss, W. Stirling, Nucl. Phys. B262, 235, (1985).

[3] CALCUL collaboration, Phys.Lett. 105B, 215 (1981).

[4] H.E. Haber,"Spin formalism and applications to new physics searches", in Proc. XXI SLAC Summer Institute on Particle Physics, ed. L. DePorcel and C. Dunwoodie, 1994, p.231.

[5] P. Cherzor, V. Ilyin, A. Pukhov, "Optimization of symbolic evaluation of helicity amplitudes", proc. of ACAT 2000 Workshop, Fermilab, October 16-20, 2000, [hep-ph/0101265].

[6] P. Cherzor, "Code generation (automatized programming) of symbolic formulae for helicity amplitudes", hep-ph/0309251.

[7] A. Denner, H. Eck, O. Hahn and J. Küblbeck. Phys. Lett. **B291** (1992) 278; Nucl. Phys. **B387** (1992) 467.

[8] R. Vega, J. Wudka, "A covariant method for calculating helicity amplitudes", Phys. Rev. D56 (1997) 6037, [hep-ph/9511318].